


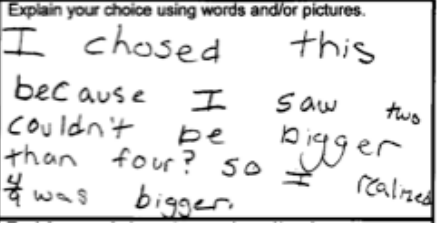
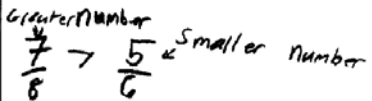
## Comparing Two Fractions Sample Student Responses

The *Comparing Two Fractions* diagnostic assessment focuses on three particular misconceptions that students have regarding how to compare fractions. Sample student responses indicative of each misconception are provided separately below, along with samples of correct student responses. To determine the degree of understanding and misunderstanding, it's important to consider both the student's answer to the selected response and the student's explanation text and representations.

### Misconception 1 (M1): Viewing a Fraction as Two Separate Numbers / Applying Whole-Number Thinking


Often, students do not perceive a fraction as a single quantity but rather see it as a pair of whole numbers, and they apply whole-number thinking by comparing the size of the numbers in the denominators or numerators or both. (For more information, go to the "Research-Based Misconceptions" tab.)

Item	Sample Student Responses with Evidence of Misconception 1	Notes
Pre-Assmt #1	<p>1.</p> <p> <input checked="" type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p> <math>\frac{5}{12}</math>      <math>\frac{3}{5}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p style="text-align: center;"> <math>\frac{5}{12} - \frac{3}{5} = \frac{2}{7}</math>            ↑         </p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The subtraction shows the student using whole-number thinking to compare the fractions</li> </ul>
Post-Assmt #1	<p>1.</p> <p> <input checked="" type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p> <math>\frac{7}{12}</math>      <math>\frac{5}{8}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p style="text-align: center;">It is greater because the denominator is bigger than the other's</p> <p style="text-align: center;">"It is greater because the denominator is bigger than the other's"</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation clearly states that the student is focusing on the size of the whole numbers in the denominators</li> </ul>
Post-Assmt #2	<p>2.</p> <p> <input type="radio"/> Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p> <math>\frac{6}{7}</math>      <math>\frac{8}{9}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p style="text-align: center;">6/7 is less than 8/9 because the 9 is bigger than the 7.</p> <p style="text-align: center;">"6/7 is less than 8/9 because the 9 is bigger than the 7."</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen (while it is the correct response, it can also indicate this misconception)</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation ("the 9 is bigger than the 7") shows that the student is comparing the denominators as whole numbers</li> </ul>

Item	Sample Student Responses with Evidence of Misconception 1	Notes
Pre-Assmt #4	<p>4.</p> <p> <input checked="" type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=) </p> <p> <math>\frac{7}{9}</math>      <math>\frac{3}{5}</math> </p> <p>  </p> <p>Explain your choice using words and/or pictures.</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen (while it is the correct response, it can also indicate this misconception)</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The circled amounts correspond to the numerators, which are being compared as whole numbers</li> </ul>
Pre-Assmt #5	<p>5.</p> <p> <input type="radio"/> Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=) </p> <p> <math>\frac{2}{5}</math>      <math>\frac{4}{9}</math> </p> <p>  </p> <p>Explain your choice using words and/or pictures.</p> <p style="text-align: center;"><b>“I chosed [chosed] this because I saw two couldn't be bigger than four? So I realized 4/9 was bigger.”</b></p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen (while it is the correct response, it can also indicate this misconception)</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation compares the numerators as whole numbers, noting that 4 is bigger than 2</li> </ul>
Pre-Assmt #6	<p>6.</p> <p> <input checked="" type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=) </p> <p> <math>\frac{7}{8}</math>      <math>\frac{5}{6}</math> </p> <p>  </p> <p>Explain your choice using words and/or pictures.</p> <p style="text-align: center;"><b>“Greater number” “Smaller number”</b></p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen (while it is the correct response, it can also indicate this misconception)</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation compares the numerators as whole numbers; the student draws arrows pointing to 7 (“greater number”) and 5 (“smaller number”)</li> </ul>

## Misconception 2 (M2): An Over-Reliance on Unit Fractions / A Focus on “Smaller Is Bigger”

Students with this misconception consistently compare only the denominators of the two given fractions. They apply what they know about unit fractions to reason that the larger the denominator, the smaller the value of the fraction (e.g., they see  $1/3$  as greater than  $3/5$ ). These students have overgeneralized the concept that “smaller is bigger” to all cases without consideration of the numerator. (For more information, go to the “Research-Based Misconceptions” tab.)

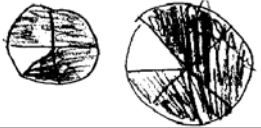
Item	Sample Student Responses with Evidence of Misconception 2	Notes
Pre-Assmt #1	<p>1.</p> <p>Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)</p> <p><math>\frac{5}{12}</math>      <math>\frac{3}{5}</math></p> <p>Explain your choice using words and/or pictures.  <math>\frac{3}{5}</math> is greater than <math>\frac{5}{12}</math> because <math>\frac{3}{5}</math> has a lower denominator.</p> <p>“<math>3/5</math> is greater than <math>5/12</math> because <math>3/5</math> has a lower denominator”</p>	<ul style="list-style-type: none"> <li>•The misconception selected response is chosen (while it is the correct response, it can also indicate this misconception)</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>•The explanation focuses on the lower denominator as the reason for the fraction being larger</li> </ul>
Pre-Assmt #2	<p>2.</p> <p>Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)</p> <p><math>\frac{4}{5}</math>      <math>\frac{6}{7}</math></p> <p>Explain your choice using words and/or pictures.  </p>	<ul style="list-style-type: none"> <li>•The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>•The diagram indicates that fifths are bigger slices and sevenths are smaller slices, suggesting that a larger denominator always results in a smaller fraction</li> </ul>
Pre-Assmt #2	<p>2.</p> <p>Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)</p> <p><math>\frac{4}{5}</math>      <math>\frac{6}{7}</math></p> <p>Explain your choice using words and/or pictures.          The smaller the bottom number the bigger the peices [pieces].</p> <p>“The smaller the bottom number the bigger the peices [pieces].”</p>	<ul style="list-style-type: none"> <li>•The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>•The explanation focuses solely on the size of the denominator, stating, “The smaller the bottom number, the bigger the pieces”</li> </ul>
Pre-Assmt #3	<p>3.</p> <p>Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)</p> <p><math>\frac{6}{8}</math>      <math>\frac{3}{4}</math></p> <p>Explain your choice using words and/or pictures.          The fraction <math>\frac{3}{4}</math> has the smallest denominator out of both fractions.</p> <p>“The fraction <math>3/4</math> has the smallest denominator out of both fractions.”</p>	<ul style="list-style-type: none"> <li>•The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>•The explanation identifies the larger fraction by focusing solely on the size of the denominators</li> </ul>


Item	Sample Student Responses with Evidence of Misconception 1	Notes
Pre-Assmt #4	<p>4.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <p>Greater than (&gt;)</p> <p>Less than (&lt;)</p> <p>Equivalent (=)</p> </div> <div style="margin-right: 20px;"> <math>\frac{7}{9}</math> </div> <div style="margin-right: 20px;"> <p>•</p> </div> <div style="margin-right: 20px;"> <math>\frac{3}{5}</math> </div> </div> <p style="text-align: center;">"I rather char [share] a candy bar with 5 pepol [people] than 9 pepol"</p>	<p>•The misconception selected response is chosen</p> <p style="text-align: center;"><b>AND</b></p> <p>•The explanation focuses solely on the size of the denominators to determine which fraction is larger, stating that a candy bar divided among 5 people will result in larger pieces than one divided among 9 people</p>
Pre-Assmt #1	<p>1.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <p>Greater than (&gt;)</p> <p>Less than (&lt;)</p> <p>Equivalent (=)</p> </div> <div style="margin-right: 20px;"> <math>\frac{5}{12}</math> </div> <div style="margin-right: 20px;"> <p>•</p> </div> <div style="margin-right: 20px;"> <math>\frac{3}{5}</math> </div> </div> <p style="text-align: center;">"Think of a chocolate bar and think which one will have bigger bricks. (Hersheys)."</p>	<p>•The misconception selected response is chosen</p> <p style="text-align: center;"><b>AND</b></p> <p>•The explanation uses the example of a candy bar and focuses on which will have "bigger bricks," indicating that the student is considering only the size of the denominator; this is further reinforced by the diagram, which shows that twelfths are smaller than fifths</p>

### Misconception 3 (M3): Numerator and Denominator Have an Additive Relationship / A Focus on the Difference from One Whole

Students with this misconception understand that it's important to pay attention to the relationship between the numerator and denominator, but they believe that this relationship is expressed through addition or subtraction. As a result, they will pay attention to the difference between the numerator and denominator in order to compare fractions.


Many students apply this reasoning only when the numerator and denominator of each fraction have a difference of one. For example, when comparing  $\frac{8}{9}$  and  $\frac{4}{5}$ , students reason that since  $\frac{8}{9}$  is only one piece away ( $\frac{1}{9}$ ) from  $\frac{9}{9}$  or one whole, and  $\frac{4}{5}$  is also one piece ( $\frac{1}{5}$ ) away from  $\frac{5}{5}$  or one whole, the two fractions must be equal, as they are each "one away" from a whole. (For more information, go to the "Research-Based Misconceptions" tab.)

Item	Sample Student Responses with Evidence of Misconception 3	Notes
Pre-Assmt #2	<p>2.</p> <p> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </p> <p> <math>\frac{4}{5}</math>      <math>\frac{6}{7}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p>both need to be filled in to a whole..</p> <p>"Both need 1 filled in to be a whole."</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation focuses on both fractions being one part away from a whole</li> </ul>
Pre-Assmt #2	<p>2.</p> <p> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </p> <p> <math>\frac{4}{5}</math>      <math>\frac{6}{7}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p>Each fraction needs to add <math>\frac{1}{5}</math> or <math>\frac{1}{7}</math> and that means they are the same.</p> <p>"Each fraction needs to add <math>\frac{1}{5}</math> or <math>\frac{1}{7}</math> and that means they are the same."</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>Though the student correctly identifies the remaining single fractional part, the explanation shows that the student sees this as meaning "they are the same"</li> </ul>
Pre-Assmt #2	<p>2.</p> <p> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </p> <p> <math>\frac{4}{5}</math>      <math>\frac{6}{7}</math> </p> <p>Explain your choice using words and/or pictures.</p>  <p>"They are both 2 fractions away from 0."</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>The diagrams highlight one piece remaining to complete a whole, illustrating this as the reason for the fractions being equivalent</li> </ul>
Pre-Assmt #4	<p>4.</p> <p> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </p> <p> <math>\frac{7}{9}</math>      <math>\frac{3}{5}</math> </p> <p>Explain your choice using words and/or pictures.</p> <p>they are both 2 fractions away from 0.</p> <p>"They are both 2 fractions away from 0."</p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation focuses on how many parts each fraction is from a benchmark (in this case 0, which is incorrect)</li> </ul>

Item	Sample Student Responses with Evidence of Misconception 1	Notes
Post- Assmt #4	<div style="border: 1px solid black; padding: 5px;"> <p>4.</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="text-align: center;"> <math>\frac{9}{11}</math> </div> <div style="text-align: center;"> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </div> <div style="text-align: center;"> <math>\frac{5}{7}</math> </div> </div> <p style="margin-top: 10px;"> <input type="radio"/> Explain your choice using words and/or pictures.  <i>Both of the top numbers plus two equal the bottom numbers</i> </p> </div> <p style="margin-top: 10px;"><b>“Both of the top numbers plus two equal the bottom numbers”</b></p>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation focuses on the difference between the numerator and denominator; since each fraction’s numerator and denominator differ by 2, the student states that they are equivalent</li> </ul>
Post- Assmt #6	<div style="border: 1px solid black; padding: 5px;"> <p>6.</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="text-align: center;"> <math>\frac{9}{10}</math> </div> <div style="text-align: center;"> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </div> <div style="text-align: center;"> <math>\frac{6}{7}</math> </div> </div> <p style="margin-top: 10px;"> <input type="radio"/> Explain your choice using words and/or pictures.  <math>\begin{array}{r} 10 \\ -9 \\ \hline 1 \end{array}</math> as <math>\begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array}</math> </p> </div>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation uses subtraction to show that the difference between the numerator and denominator in each case is the same; the student disregards the size of the fractional pieces</li> </ul>
Pre- Assmt #6	<div style="border: 1px solid black; padding: 5px;"> <p>6.</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="text-align: center;"> <math>\frac{7}{8}</math> </div> <div style="text-align: center;"> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </div> <div style="text-align: center;"> <math>\frac{5}{6}</math> </div> </div> <div style="margin-top: 10px;"> <input type="radio"/> Explain your choice using words and/or pictures.   </div> </div>	<ul style="list-style-type: none"> <li>The misconception selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The explanation shows both fractions drawn with equal-size wholes, and vertical lines drawn to highlight the one part remaining on the right, illustrating this as the reason the fractions are equivalent</li> </ul>

## Incorrect Reasoning That Is Not One of These Misconceptions

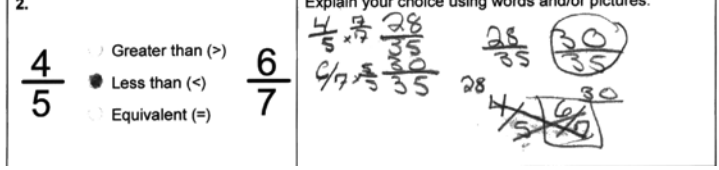
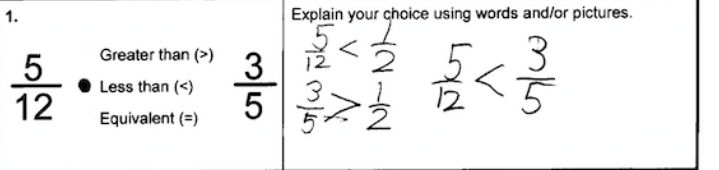
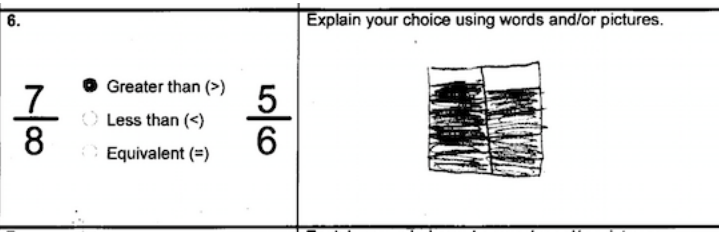
There may be some cases in which the student selects the response that indicates the misconception but does not provide convincing evidence that he or she actually has the misconception. In some cases, the student may have a different set of difficulties than the specific misconceptions targeted by this probe. Here are two examples:

Item	Sample Student Responses with Evidence of Correct Responses	Notes
Pre-Assmt #2	<p>2.</p> <div style="display: flex; align-items: center; gap: 20px;"> <div style="text-align: center;"> <math>\frac{4}{5}</math> </div> <div style="text-align: center;"> <p>Greater than (&gt;)</p> <p>Less than (&lt;)</p> <p><input checked="" type="radio"/> Equivalent (=)</p> </div> <div style="text-align: center;"> <math>\frac{6}{7}</math> </div> </div> <p style="margin-top: 10px;">"I chose equivalent because 4 + 2 equals 6 and 5 + 2 equals 7."</p>	<ul style="list-style-type: none"> <li>The M3 selected response is chosen</li> </ul> <p style="text-align: center;"><b>HOWEVER</b></p> <ul style="list-style-type: none"> <li>The explanation does not fit M3—the student sees the fractions as equivalent because both numerators and denominators differ from the other by 2 (a student with M3 would instead focus on both fractions being “1 away” from a whole)</li> </ul>
Pre-Assmt #4	<p>4.</p> <div style="display: flex; align-items: center; gap: 20px;"> <div style="text-align: center;"> <math>\frac{7}{9}</math> </div> <div style="text-align: center;"> <p>Greater than (&gt;)</p> <p><input checked="" type="radio"/> Less than (&lt;)</p> <p>Equivalent (=)</p> </div> <div style="text-align: center;"> <math>\frac{3}{5}</math> </div> </div> <div style="margin-top: 10px;"> <p>Explain your choice using words and/or pictures.</p>  </div>	<ul style="list-style-type: none"> <li>The M2 selected response is chosen</li> </ul> <p style="text-align: center;"><b>HOWEVER</b></p> <ul style="list-style-type: none"> <li>The explanation does not fit M2—the student tries to compare the fractions by drawing each one, but the drawings are not accurate (a student with M2 would instead focus on ninths being smaller than fifths)</li> </ul>

## Correct Reasoning

Students with correct reasoning about comparing fractions are often able to do one or more of the following:

- Multiply the numerator and denominator by a common factor to get common denominators
- Use benchmarks (e.g., understand that  $\frac{5}{12}$  is less than  $\frac{1}{2}$ , but  $\frac{3}{5}$  is greater than  $\frac{1}{2}$ )
- Use accurate diagrams to compare the fractions

Item	Sample Student Responses with Correct Reasoning	Notes
Pre-Assmt #2	<p>2.</p> <p> <input type="radio"/> Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p><math>\frac{4}{5}</math>      <math>\frac{6}{7}</math></p> <p>Explain your choice using words and/or pictures.</p> 	<ul style="list-style-type: none"> <li>• The correct selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>• The fractions are correctly converted into equivalent fractions with common denominators</li> </ul>
Pre-Assmt #1	<p>1.</p> <p> <input type="radio"/> Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p><math>\frac{5}{12}</math>      <math>\frac{3}{5}</math></p> <p>Explain your choice using words and/or pictures.</p> 	<ul style="list-style-type: none"> <li>• The correct selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>• The explanation shows how each fraction compares to the benchmark of <math>\frac{1}{2}</math></li> </ul>
Pre-Assmt #6	<p>6.</p> <p> <input checked="" type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </p> <p><math>\frac{7}{8}</math>      <math>\frac{5}{6}</math></p> <p>Explain your choice using words and/or pictures.</p> 	<ul style="list-style-type: none"> <li>• The correct selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>• The diagram shows equal-size wholes &lt;&lt;correctly partitioned and&gt;&gt;being compared visually</li> </ul>
Pre-Assmt #7	<p>7.</p> <p> <input type="radio"/> Greater than (&gt;)  <input type="radio"/> Less than (&lt;)  <input checked="" type="radio"/> Equivalent (=)         </p> <p><math>\frac{4}{7}</math>      <math>\frac{12}{21}</math></p> <p>Explain your choice using words and/or pictures.</p> <p>3 times bigger same exact thing.</p> <p style="text-align: center;"><b>"3 times bigger, same exact thing."</b></p>	<ul style="list-style-type: none"> <li>• The correct selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>• The explanation shows that the student sees the second fraction as "3 times bigger," and this is the reason they are equivalent</li> </ul>



Item	Sample Student Responses with Correct Reasoning	Notes
Post-Assmt #2	<p>2.</p> <div style="display: flex; align-items: center; justify-content: space-between;"> <div style="text-align: center;"> <math>\frac{6}{7}</math> </div> <div style="text-align: center;"> <input type="radio"/> Greater than (&gt;)  <input checked="" type="radio"/> Less than (&lt;)  <input type="radio"/> Equivalent (=)         </div> <div style="text-align: center;"> <math>\frac{8}{9}</math> </div> </div> <p style="text-align: center;"><b>"<math>1/7 &gt; 1/9</math>, so <math>6/7</math> (<math>7/7 - 1/7</math>) is less than <math>8/9</math> (<math>9/9 - 1/9</math>)."</b></p>	<ul style="list-style-type: none"> <li>The correct selected response is chosen</li> </ul> <p style="text-align: center;"><b>AND</b></p> <ul style="list-style-type: none"> <li>The student notices that <math>6/7</math> is <math>1/7</math> away from 1 and that <math>8/9</math> is <math>1/9</math> away from 1, and reasons that since <math>1/7 &gt; 1/9</math>, <math>8/9</math> is greater because the difference between the fraction and 1 is less</li> </ul>