Fractions: Adding and Subtracting

The Fractions: Adding and Subtracting assessment is designed to elicit information about several common misconceptions that students have when adding or subtracting two fractions:

» Misconception 1: Adding Numerators and/or Adding Denominators
» Misconception 2: Common Denominators with Incorrect Numerators

Although you can access the assessment here at any time, we strongly recommend that you reference the information below to learn more about this misconception, including how it appears in student work, and how to score pre- and post-assessments once you have given them to students.

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Topic Background

Learn about comparing decimals with embedded or trailing zeros

Students need to begin with a foundational understanding of a fraction representing a part of some whole, and how the numerator represents a number of parts, while the denominator represents the size of the parts. This understanding is developed further as students learn to represent fractions with different visual models, and to connect parts of the visual models with the numerator or denominator.

Students also need to develop an understanding of a fraction with a numerator larger than 1 as the sum or difference of unit fractions with the same denominator. For example, \( \frac{3}{4} \) is not only viewed as 3 parts out of 4, but also as adding three \( \frac{1}{4} \)'s, or \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \). Similarly, \( \frac{5}{7} \) could be viewed as \( \frac{7}{7} - \frac{1}{7} - \frac{1}{7} \).

As students develop their understanding of fraction addition and subtraction, they learn to view each operation as adding or subtracting the number of parts (numerator), and not the denominator, since it represents the size of the parts, not a count of the parts.

Connections to Common Core Standards in Mathematics (CCSS)

The CCSS outline specific understandings that students should be able to meet at each grade level.

At grade 4, students should be able to extend their understanding of addition and subtraction from whole numbers to fractions, and different ways that addition and subtraction of fractions can be represented.

**Develop understanding of fractions \( \frac{a}{b} \) with \( a > 1 \) as the sum of fractions \( \frac{1}{b} \).**

- **4.NF.3a.** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **4.NF.3d.** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

At grade 5, students should be able to use the concept of *equivalence* to extend their prior understandings of fraction addition and subtraction.

**Use equivalent fractions as a strategy to add and subtract fractions.**

- **5.NF.1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, \( \frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12} \). (In general, \( \frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd} \).)*
- **5.NF.2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result \( \frac{2}{5} + \frac{1}{2} = \frac{3}{7} \), by observing that \( \frac{3}{7} < \frac{1}{2} \).*
Student Misconceptions

Learn about student misconceptions related to the topic.

When students are developing the understandings described above (see Topic Background), they can develop flawed understanding leading to misconceptions about addition and subtraction of fractions. Once students have been introduced to a variety of strategies to use to assist with adding and subtracting, many overgeneralize, confuse, or misapply strategies. Two particular misconceptions, noted in the research on students’ mathematical reasoning about fraction addition and subtraction, are the focus of this diagnostic assessment.

The following common misunderstandings and misconceptions that occur when adding and subtracting fractions are targeted in the *Fractions: Adding and Subtracting* assessments:

- **Misconception 1 (M1): Adding Numerators and/or Adding Denominators.** Students apply whole-number reasoning by adding either the denominators or the numerators to determine the sum.

- **Misconception 2 (M2): Common Denominators with Incorrect Numerators.** Students apply a partial algorithm approach where they multiply the denominators but fail to change the numerators to maintain equivalence.

Watch the brief video clip for a fuller description of this misconception. See included files.

To see additional examples of student work illustrating this misconception, refer to page 35 of this document.
Student Misconceptions

Resources


Administering the Pre-Assessment

Learn how to introduce the pre-assessment to your students.

About This Assessment

These EM2 diagnostic formative pre- and post-assessments are composed of items with specific attributes associated with student conceptions that are specific to adding and subtracting fractions. Each of the items within any EM2 assessment includes a selected response (multiple choice) and an explanation component.

This assessment targets proper, non-unit fractions due to the particular difficulties that these pairs elicit, as identified in the mathematics research listed in the section "Research-Based Misconceptions." The fractions being added or subtracted in this assessment are confined to:

- Proper fractions - A proper fraction is a fraction where the numerator (top number) is less than the denominator (the bottom number)
- Proper fractions with denominators less than or equal to 9

The learning target for the Fractions: Adding and Subtracting assessment is as follows:

The learner will accurately add or subtract two fractions with different numerators and different denominators.

Prior to Giving the Pre-Assessment

- Arrange for 15 minutes of class time to complete the administration process, including discussing instructions and student work time. Since the pre-assessment is designed to elicit misconceptions before instruction, you do not need to do any special review of this topic before administering the assessment.

Administering the Pre-Assessment

- Inform students about the assessment by reading the following:

Today you will complete a short individual activity, which is designed to help me understand how you think about adding and subtracting fractions.

- Distribute the assessment and read the following:

The activity includes 6 problems. For each problem, choose your answer by completely filling in the circle to show which answer you think is correct. Because the goal of the activity is to learn more about how you think about adding or subtracting fractions, it's important for you to include some kind of explanation in the space provided. This can be a picture or words, or a combination of pictures and words that shows how you chose your answer.

You will have about 15 minutes to complete all the problems. When you are finished please place the paper on your desk and quietly [read, work on ___] until everyone is finished.
Administering the Pre-Assessment

- Monitor the students as they work on the assessment, making sure that they understand the directions. Although this is not a strictly timed assessment, it is designed to be completed within a 15-minute timeframe. Students may have more time if needed. When a few minutes remain, say:

  You have a few minutes left to finish the activity. Please use this time to make sure that all of your answers are as complete as possible. When you are done, please place the paper face down on your desk. Thank you for working on this activity today.

- Collect the assessments.

After Administering the Pre-Assessment

Use the analysis process (found in the Scoring Guide PDF document under the Scoring Process section) to analyze whether your students have either or both of two possible misconceptions:

- **Misconception 1: Adding Numerators and/or Adding Denominators**
- **Misconception 2: Common Denominators with Incorrect Numerators**
Scoring

Learn about the scoring process by reviewing the Scoring Guide.

The Fractions: Adding and Subtracting assessment is composed of 6 items with specific attributes associated with different misconceptions that are directly related to adding or subtracting fractions. We encourage you to carefully read the Scoring Guide to understand these specific attributes and to find information about analyzing your students’ responses.

The scoring guide is intended for use with both the pre-assessment and the post-assessment for Fractions: Adding and Subtracting.

To use this guide, we recommend following these steps:

• Read the Misconception Description above, and be sure you understand what the misconceptions are. You may want to view the video description of the misconceptions found in the included files. Numerous examples of student work illustrating this misconception are included in this guide.
• Familiarize yourself with the six assessment items and what they are assessing;
• Consider completing the optional scoring practice items and checking yourself with the answer key;
• Score your students’ work using the “Analysis Process” described below;
• Refer to the various examples found here and in the Sample Student Responses for guidance when you are unsure about the scoring.
### Scoring

**PRE-ASSESSMENT**

The pre-assessment is composed of six items with specific attributes associated with understandings and misunderstandings related to adding and subtracting fractions.

<table>
<thead>
<tr>
<th>Item</th>
<th>Understandings and Misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Item 1</strong></td>
<td></td>
</tr>
</tbody>
</table>
1. \(\frac{1}{3} + \frac{1}{4} = \frac{2}{7}\)  
2. \(\frac{2}{12}\)  
3. \(\frac{7}{12}\)  

Correct Response: \(\frac{7}{12}\)  

Item may elicit evidence of:  
- *Incorrectly using whole-number thinking to add the fractions:* Students with Misconception 1 will reason that \(1 + 1 = 2\) and \(3 + 4 = 7\), so the sum is \(2/7\).  
- *An inability to create equivalent fractions* (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 12, and that \(1 + 1 = 2\), so the sum is \(2/12\).*  |
| **Item 2** |  
1. \(\frac{2}{3} + \frac{1}{4} = \frac{11}{12}\)  
2. \(\frac{3}{12}\)  
3. \(\frac{3}{7}\)  

Correct Response: \(\frac{11}{12}\)  

Item may elicit evidence of:  
- *Incorrectly using whole-number thinking to add the fractions:* Students with Misconception 1 will reason that \(2 + 1 = 3\) and \(3 + 4 = 7\), so the sum is \(3/7\).  
- *An inability to create equivalent fractions* (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 12, and that \(2 + 1 = 3\), so the sum is \(3/12\).*  |
| **Item 3** |  
1. \(\frac{2}{9} + \frac{3}{5} = \frac{5}{45}\)  
2. \(\frac{5}{14}\)  
3. \(\frac{37}{45}\)  

Correct Response: \(\frac{37}{45}\)  

Item may elicit evidence of:  
- *Incorrectly using whole-number thinking to add the fractions:* Students with Misconception 1 will reason that \(2 + 3 = 5\) and \(9 + 5 = 14\), so the sum is \(5/14\).  
- *An inability to create equivalent fractions* (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 45, and that \(2 + 3 = 5\), so the sum is \(5/45\).* |
### Scoring

**Item 4**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{2}{24}$</td>
</tr>
<tr>
<td>$\frac{7}{12}$</td>
<td>$\frac{2}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

Correct Response: $\frac{7}{12}$

**Item may elicit evidence of:**

- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $3 - 1 = 2$. When faced with $4 - 6$, some students will assume it should be positive and will conclude the denominator is 2. Others may correctly calculate the difference as -2, but being unaccustomed to seeing negative numbers in fractions, will just make it positive. They will conclude that the sum is $2/2$.

- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 24, and that $3 - 1 = 2$, so the difference is $2/24$.

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**Item 5**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$\frac{7}{8}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{6}{4}$</td>
</tr>
<tr>
<td>$\frac{6}{32}$</td>
<td>$\frac{5}{8}$</td>
<td></td>
</tr>
</tbody>
</table>

Correct Response: $\frac{5}{8}$

**Item may elicit evidence of:**

- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $7 - 1 = 6$ and $8 - 4 = 4$, so the difference is $6/4$.

- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 32, and that $7 - 1 = 6$, so the difference is $6/32$.

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**Item 6**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{24}$</td>
</tr>
<tr>
<td>$\frac{2}{5}$</td>
<td>$\frac{2}{24}$</td>
<td></td>
</tr>
</tbody>
</table>

Correct Response: $\frac{1}{24}$

**Item may elicit evidence of:**

- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $3 - 1 = 2$ and $8 - 3 = 5$, so the difference is $2/5$.

- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 24, and that $3 - 1 = 2$, so the difference is $2/24$.

If students choose an incorrect response that does not indicate M1 or M1 thinking, review their explanations to determine what difficulty they are having.
Scoring

Pre-Assessment Analysis Process

Some important things to know about the analysis process for this diagnostic assessment:

- This diagnostic assessment has been validated to reliably predict the likelihood that a student has one or both of Misconception 1 and Misconception 2. You can weigh the relative likelihood that your student has either of these misconceptions by considering whether the student’s written response provides “Strong Evidence” or “Weak Evidence” of the misconception.

- If a student is determined to show evidence of the misconception on even just one of items, the students is likely to have the misconception.

- For each item, you need to look both at the selected response choice as well as the explanation. Students will show evidence of the misconception only if they select the corresponding response choice and have an explanation that supports the misconception. To learn more about how to tell whether an explanation supports one of the misconceptions, go to the “Student Misconceptions” on page 3 and watch the video provided.

- An optional scoring guide template is provided for your use when you score your own students’ diagnostic assessments. In each row of the assessment, write the name of one of your students. Then circle the appropriate information for each item on the pre-assessment (shaded) and later the post-assessment (in white).

HOW TO DETERMINE IF A STUDENT HAS ONE OR MORE OF THE MISCONCEPTIONS:

1. For each item, use the table below to determine what the selected response might indicate.

   For example, let’s say a student responds 2/7 for item 1. Looking at the table, we see that 2/7 might indicate the presence of M1.

```
<table>
<thead>
<tr>
<th>Item #</th>
<th>Correct</th>
<th>M1 Likely Responses</th>
<th>M2 Likely Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{7}{12})</td>
<td>2 (\frac{2}{7})</td>
<td>2 (\frac{2}{12})</td>
</tr>
</tbody>
</table>
```

You can use the table below to determine whether the student’s selected response indicates the possibility of M1, M2 or of correct thinking.
Scoring

Table 1. Response Patterns for the Pre-Assessment

<table>
<thead>
<tr>
<th>Item #</th>
<th>Correct</th>
<th>M1 Likely Response</th>
<th>M2 Likely Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{7}{12})</td>
<td>(\frac{2}{7})</td>
<td>(\frac{2}{12})</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{11}{12})</td>
<td>(\frac{3}{7})</td>
<td>(\frac{3}{12})</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{37}{45})</td>
<td>(\frac{5}{14})</td>
<td>(\frac{5}{45})</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{7}{12})</td>
<td>(\frac{2}{2})</td>
<td>(\frac{2}{24})</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{5}{8})</td>
<td>(\frac{6}{4})</td>
<td>(\frac{6}{32})</td>
</tr>
<tr>
<td>6</td>
<td>(\frac{1}{24})</td>
<td>(\frac{2}{5})</td>
<td>(\frac{2}{24})</td>
</tr>
</tbody>
</table>

What if there’s no multiple-choice response selected?
In that case, carefully consider the explanation the student gives. If the explanation leaves no doubt which of the fractions the student would have selected and no doubt about how the student is reasoning, you can code it Correct, M1, or M2 with “Strong Evidence” of the appropriate misconception. (For additional guidance on determining the strength of the evidence, see the “What counts…” information provided in Step 2 below.)

However, if the explanation leaves some question about what the student was thinking, code it as “Other” and move on to the next question.

2. For each item, carefully consider the student’s explanation to determine what the response indicates, and note whether the evidence from the explanation is strong or weak.

To continue the example above, even though the student selected 2/7, the student may or may not have used M1 reasoning to come up with 2/7.

<table>
<thead>
<tr>
<th>Item #</th>
<th>Correct</th>
<th>M1 Likely Responses</th>
<th>M2 Likely Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{7}{12})</td>
<td>(\frac{2}{7})</td>
<td>(\frac{2}{12})</td>
</tr>
</tbody>
</table>

Therefore, it is particularly important to also consider the student’s explanation in order to determine which of the misconceptions is present.
Scoring

A Caution!
The table of response patterns above shows that each response indicates only one possibility; for example, a response of 2/7 for item 1 only indicates the possibility of M1. However, it is still necessary to check the student's explanation to confirm evidence of the misconception. It is not unusual for a student to provide a response that appears to point to a particular misconception, but the explanation may be contradictory, or the student may have arrived at that response using different reasoning than what's expected for the misconception.
The upshot: You can't simply rely on the selected response. Always check the explanation along with the selected response choice.

An explanation can be categorized as "Strong Evidence" of a misconception, "Weak Evidence" of a misconception, or "No Evidence" of a misconception.

What counts as "Strong Evidence" of a misconception in the pre-assessment?

In general, responses with strong evidence of a misconception include a clear indication that the student is exhibiting the reasoning typical for that misconception. There is no need to make inferences about what the student is thinking; that thinking is quite clear from the combination of the selected response and the explanation. Below are two examples of student responses with strong evidence of a misconception, using pre-assessment items. To see additional examples of student responses that illustrate these misconceptions, go to the "Sample Student Responses" on page 36.

Example A – Strong Evidence of M1
For M1, the explanation will include clear evidence that the student is reasoning about the addition of the numbers in the fraction as if they were separate whole numbers. (For a more detailed description of this misconception, see the video explanations of each misconception)

![Image of fraction addition problem]

For item 1, this student chose 2/7, which could indicate either M1 (see table above). Looking at the explanation, there is clear evidence that the student is using whole-number reasoning because the student shows the addition of the numerators as well as the addition of the denominators. Because it is so clear that the student is using whole-number reasoning, this is “Strong Evidence” of M1.

Example B – Strong Evidence of M2
For M2, the explanation will include clear evidence that the student is multiplying to find common denominators, and adding the numerators without first finding equivalent fractions. (For a more detailed description of this misconception, see the video explanations of each misconception.)
Scoring

For item 2, this student chose 3/12, which might indicate M2 (see table above). Looking at the explanation, there is clear evidence that the student is multiplying the denominators to establish a common denominator, then adding the numerators. This is “Strong Evidence” of M2.

Can a “Correct” response be considered to have “Strong Evidence?”

Yes, a correct response can also have “Strong Evidence,” “Weak Evidence,” or “No Evidence” as well. While it is not necessary to categorize correct responses as strong, weak or non-existent for the purposes of this diagnostic assessment, you may want to note this on your scoring template for your own purposes.

What counts as “Weak Evidence” of a misconception in the pre-assessment?

Responses with weak evidence of a misconception include some indication that the student is exhibiting the reasoning typical for that misconception. However, these responses also generally require making more inferences about what the student was thinking, or they leave some question or doubt about whether the misconception is present or to what degree it is present. Below are two examples of student responses with weak evidence of a misconception, using pre-assessment items. To see additional examples of student responses that illustrate these misconceptions, go to the “Sample Student Responses” on page 36.

Example A – Weak Evidence of M1

For item 1, this student chose 7/13, which indicates the possibility of M1 (see table above). In the explanation, the student simply repeats the problem. It is likely that they simply added numerators and denominators, but because it is not clearly spelled out, this is considered “Weak Evidence” of M1.
Scoring

Example B – Weak Evidence of M2

For item 4, this student chose 2/24, which indicates the possibility of M2 (see table above). Again, in the explanation, the student simply repeats the problem. It is likely that they simply multiplied to find the common denominator, then just subtracted the numerators, but because it is not clearly spelled out, this is considered “Weak Evidence” of M2.

What if the student selects one of the choices, but provides no explanation?
If a student selects an M1 or M2 response choice but provides no explanation at all, this is not considered convincing evidence of the misconception, and can be scored as “Other” on the scoring template.

What if the student’s choice matches a misconception, but the explanation does not support it?
If a student’s response choice suggests a possible misconception, but the student’s explanation does not support it, then the item is not considered to be indicative of the misconception, and again can be scored as “Other.”

3. After you have analyzed each of the individual items for a student, use the guidelines below to determine whether the student has either of the misconceptions.

This diagnostic assessment has been validated to predict the possible presence of M1 or M2 for a student. If a student is determined to show evidence of the misconception on even just one of the items, the students is likely to have that misconception, regardless of whether the evidence is coded as “Strong” or “Weak.” The relative number of items with weak or strong evidence gives you information about how strongly the misconception may be present for the student.

What if my student has only one item indicating one of the misconceptions with “Weak Evidence,” and the rest are correct?
Even if your student has only one item with “Weak Evidence” of a misconception, this diagnostic assessment is validated to predict that it is likely your student may have that misconception. However, the presence of only one item with weak evidence suggests that the misconception may not be very deeply rooted in this student’s thinking.

You may want to keep an eye on this student during regular classwork to watch for other evidence of this misconception.

What if the student’s explanation is contradictory to the multiple-choice response chosen?
If you come across a response in which the explanation seems to contradict the response choice, it is considered a possible indication of the misconception. Look for additional evidence, either on these assessments or from the student’s comments in class.
Scoring

(Optional) Scoring Practice Items—Pre-Assessment

The following sample student responses are provided as an optional practice set. If you would like to practice scoring several items to further clarify your understanding of the scoring process, you may try scoring the following 10 items.

We recommend scoring one or two at a time, and checking your scoring as you go against our key found on page 18.

Practice Example 1

I got 5/8 because $4 \times \frac{2}{8} = 8$, so if the top fraction was 7/8, it would stay the same.

Practice Example 2

“If you subtract [subtract] 3/8 and 1/3, you would get 2/5.”

Practice Example 3
### Scoring

#### Practice Example 4

```
9 \frac{2}{5} + \frac{3}{4} = \frac{5}{14} \quad \frac{27}{45}
```

“*You times the denominators by the numerator.*”

#### Practice Example 5

```
0 \frac{3}{4} - \frac{1}{3} = \frac{2}{24}
```

“I just did $8 \times 3$ equal 24 than [then] I did $3 - 1$ equals $2$ and then I got $\frac{2}{24}$ for my answer.”

#### Practice Example 6

```
\frac{3}{4} - \frac{1}{6} = \frac{2}{24} \quad \frac{7}{12} \quad \frac{2}{2}
```

#### Practice Example 7

```
\frac{2}{9} \frac{3}{5} = \frac{18}{45} \quad \frac{45}{45}
```

“I just did $8 \times 3$ equal 24 than [then] I did $3 - 1$ equals $2$ and than [then] I got $2/24$ for my answer.”
Scoring

Practice Example 8

5. \[
\begin{array}{c}
\frac{7}{8} - \frac{1}{4} = \frac{6}{32}
\end{array}
\]

Show and explain your work here.

Practice Example 9

6. \[
\begin{array}{c}
\frac{3}{8} - \frac{1}{3} = \frac{1}{24}
\end{array}
\]

Show and explain your work here.

Practice Example 10

1. \[
\begin{array}{c}
\frac{1}{3} + \frac{1}{4} = \frac{7}{12}
\end{array}
\]

Show and explain your work here.

"So you do is 1 + 1 and you get 2 and then you do 3 + 4 and you get 7 so your aser [answer] is 2/7."
Scoring

Scoring Practice Items Answer Key—Pre-Assessment

Practice Example 1

This is a “Correct” example with “Strong Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student correctly finds equivalent fractions with common denominators and subtracts correctly.

Practice Example 2

This is an example of M2 with “Strong Evidence.” The student clearly shows finding a common denominator by finding the least common multiple. However, the student then adds the numerators without first finding equivalent fractions.

Practice Example 3

This is an example of M1 with “Strong Evidence.” The student does not write out his or her steps to show subtracting the numerators or denominators; however, given the three choices, there is little doubt that the student is simply subtracting numerators and denominators.

“I got 5/8 because 4 x 2 = 8, so if the top fraction was 7/8, it would stay the same.”

“If you subtract [subtract] 3/8 and 1/3, you would get 2/5.”
Scoring

Practice Example 4

This is a “Correct” example with “Weak Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response (37/45). The student also says “You times the denominators by the numerator;” although this suggests a correct algorithm, we have to make some assumptions about what the student means, making it “Weak Evidence” that the student is reasoning correctly.

Practice Example 5

This is an example of M2 with “Strong Evidence.” The student’s explanation clearly describes multiplying the denominators to find a common denominator, then subtracting the existing numerators.

Practice Example 6

This is an example of “Other.” Although the student has selected the M2 response (2/24), the explanation does not support the typical “M2” line of reasoning. The explanation does not show that the student is multiplying denominators to get a common denominator, then subtracting the existing numerators. Instead, the student finds correct equivalent fractions, appears to subtract them correctly, but then selects 2/24. If a student’s response choice suggests a possible misconception, but the student’s explanation does not support it, then the item is not considered to be indicative of the misconception.
Scoring

Practice Example 7

This is a “Correct” example with “Strong Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student correctly finds equivalent fractions with common denominators and adds correctly.

Practice Example 8

This is an example of M2 with “Weak Evidence.” Although the student has selected the M2 response (6/32), the explanation doesn’t show that the student is necessarily finding a common denominator, then subtracting the existing numerators. This student finds correct equivalent fractions with common denominators, but it’s then unclear how the student ends up with a numerator of 6. The lack of clarity about the student’s thinking makes it “Weak Evidence” of M2.

Practice Example 9

This is an example of M1 with “Weak Evidence.” Although the student has selected the M1 response (2/5), it’s not clear that the student is simply subtracting the numerators and subtracting the denominators. This student writes 3/5 next to 3/8, and 1/5 next to 1/3, and appears to be trying to create fractions with a common denominator of 5. The lack of clarity about the student’s thinking makes it “Weak Evidence” of M1.
Scoring

Practice Example 10

This is an example of M1 with “Strong Evidence.” The student’s explanation clearly shows the student adding 1+1 to get 2 and 3+4 to get 7, and the explanation leaves no doubt that this is how the student is reasoning.

“So you do is 1 + 1 and you get 2 and then you do 3 + 4 and you get 7 so your aser [answer] is 2/7.”
Post-Assessment Items

The post-assessment is structured exactly the same as the pre-assessment, with six items with specific attributes associated with understandings and misunderstandings related to adding and subtracting fractions.

<table>
<thead>
<tr>
<th>Item</th>
<th>Understandings and Misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>Item may elicit evidence of:</td>
</tr>
<tr>
<td></td>
<td>• <em>Incorrectly using whole-number thinking to add the fractions</em>: Students with Misconception 1 will reason that $1 + 1 = 2$ and $4 + 5 = 9$, so the sum is $2/9$.</td>
</tr>
<tr>
<td></td>
<td>• <em>An inability to create equivalent fractions</em> (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is $20$, and that $1 + 1 = 2$, so the sum is $2/20$.</td>
</tr>
<tr>
<td>$\frac{1}{4} + \frac{1}{5} = ?$</td>
<td>Correct Response: $\frac{9}{20}$</td>
</tr>
<tr>
<td>$\frac{2}{9}$</td>
<td>$\frac{2}{20}$</td>
</tr>
<tr>
<td>Item 2</td>
<td>Item may elicit evidence of:</td>
</tr>
<tr>
<td></td>
<td>• <em>Incorrectly using whole-number thinking to add the fractions</em>: Students with Misconception 1 will reason that $2 + 1 = 3$ and $5 + 3 = 8$, so the sum is $3/8$.</td>
</tr>
<tr>
<td></td>
<td>• <em>An inability to create equivalent fractions</em> (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is $15$, and that $2 + 1 = 3$, so the sum is $3/15$.</td>
</tr>
<tr>
<td>$\frac{2}{5} + \frac{1}{3} = ?$</td>
<td>Correct Response: $\frac{11}{15}$</td>
</tr>
<tr>
<td>$\frac{3}{15}$</td>
<td>$\frac{3}{5}$</td>
</tr>
<tr>
<td>Item 3</td>
<td>Item may elicit evidence of:</td>
</tr>
<tr>
<td></td>
<td>• <em>Incorrectly using whole-number thinking to add the fractions</em>: Students with Misconception 1 will reason that $3 + 4 = 7$ and $8 + 5 = 13$, so the sum is $7/13$.</td>
</tr>
<tr>
<td></td>
<td>• <em>An inability to create equivalent fractions</em> (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is $40$, and that $3 + 4 = 7$, so the sum is $7/40$.</td>
</tr>
<tr>
<td>$\frac{3}{8} + \frac{4}{5} = ?$</td>
<td>Correct Response: $\frac{47}{40}$</td>
</tr>
<tr>
<td>$\frac{7}{13}$</td>
<td>$\frac{47}{40}$</td>
</tr>
</tbody>
</table>
### Scoring

<table>
<thead>
<tr>
<th>Item 4</th>
<th>Correct Response: $\frac{7}{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4}{5} - \frac{1}{3} = $</td>
<td>$\frac{3}{15}$&lt;br&gt;$\frac{7}{15}$&lt;br&gt;$\frac{3}{2}$</td>
</tr>
</tbody>
</table>

Item may elicit evidence of:
- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $4 - 1 = 3$ and $5 - 3 = 2$, so the difference is $3/2$.
- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 15, and that $4 - 1 = 3$, so the difference is $3/15$.

<table>
<thead>
<tr>
<th>Item 5</th>
<th>Correct Response: $\frac{19}{30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5}{6} - \frac{1}{5} = $</td>
<td>$\frac{4}{1}$&lt;br&gt;$\frac{4}{30}$&lt;br&gt;$\frac{18}{30}$</td>
</tr>
</tbody>
</table>

Item may elicit evidence of:
- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $5 - 1 = 4$ and $6 - 5 = 1$, so the difference is $4/1$.
- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 30, and that $5 - 1 = 4$, so the difference is $4/30$.

<table>
<thead>
<tr>
<th>Item 6</th>
<th>Correct Response: $\frac{13}{28}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{5}{7} - \frac{1}{4} = $</td>
<td>$\frac{13}{28}$&lt;br&gt;$\frac{4}{3}$&lt;br&gt;$\frac{4}{28}$</td>
</tr>
</tbody>
</table>

Item may elicit evidence of:
- **Incorrectly using whole-number thinking to add the fractions:** Students with Misconception 1 will reason that $5 - 1 = 4$ and $7 - 4 = 3$, so the difference is $4/3$.
- **An inability to create equivalent fractions** (i.e. students will find a common denominator, but not find the correct corresponding numerator; they will keep the current numerator). Students who have Misconception 2 will reason that the common denominator is 28, and that $5 - 1 = 4$, so the difference is $4/28$. 
### Scoring

<table>
<thead>
<tr>
<th>0.340</th>
<th>0.34</th>
<th>Equivalent (=)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greater than (&gt;)</td>
<td>Less than (&lt;)</td>
<td>Equivalent (=)</td>
</tr>
</tbody>
</table>

**Correct Response:** Equivalent (=)

- This item provides evidence of Misconception 3: Trailing Zeros Do Matter.
- Students with misconception 3 will reason that 0.340 > 0.34 because 340 > 34.
- Students who are only considering the numbers to the left of the decimal may also select “equivalent.”

<table>
<thead>
<tr>
<th>4.06</th>
<th>4.2</th>
<th>Equivalent (=)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greater than (&gt;)</td>
<td>Less than (&lt;)</td>
<td>Equivalent (=)</td>
</tr>
</tbody>
</table>

**Correct Response:** Less than (<)

- This item provides evidence of Misconception 2: Embedded Zeros Don’t Matter.
- Students with misconception 2 will reason that 4.06 > 4.2 because 6 > 2.
- Students who are only considering the numbers to the left of the decimal will select “equivalent.”
Scoring

Post-Assessment Analysis Process

You may want to review the bulleted items listed under “Some important things to know about the analysis process for this diagnostic assessment,” starting on page 10 under the heading, “Pre-Assessment Analysis Process.”

HOW TO DETERMINE IF A STUDENT HAS THE MISCONCEPTION:
The post-assessment uses the same scoring process as the pre-assessment. If you are not already familiar with the steps for scoring the assessment, please review that section starting on page 7.

1. For each item, use the table below to determine what the selected response might indicate.

Table 2. Response Patterns for the Post-Assessment

<table>
<thead>
<tr>
<th>Item #</th>
<th>Correct</th>
<th>M1 Likely Response</th>
<th>M2 Likely Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9/20</td>
<td>2/9</td>
<td>2/20</td>
</tr>
<tr>
<td>2</td>
<td>11/15</td>
<td>3/8</td>
<td>3/15</td>
</tr>
<tr>
<td>3</td>
<td>47/40</td>
<td>7/13</td>
<td>7/40</td>
</tr>
<tr>
<td>4</td>
<td>7/15</td>
<td>3/2</td>
<td>3/15</td>
</tr>
<tr>
<td>5</td>
<td>19/30</td>
<td>4/1</td>
<td>4/30</td>
</tr>
<tr>
<td>6</td>
<td>13/28</td>
<td>4/3</td>
<td>4/28</td>
</tr>
</tbody>
</table>

What if there’s no multiple-choice response selected?
In that case, carefully consider the explanation the student gives. If the explanation leaves no doubt which of the fractions the student would have selected and no doubt about how the student is reasoning, you can code it Correct, M1, or M2 with “Strong Evidence” of the appropriate misconception. (For additional guidance on determining the strength of the evidence, see the “What counts…” information provided in Step 2 below.)

However, if the explanation leaves some question about what the student was thinking, code it as “Other” and move on to the next question.

2. For each item, carefully consider the student’s explanation to determine what the response indicates, and note whether the evidence from the explanation is strong or weak.

Recall that it is particularly important to also consider the student’s explanation in order to determine which of the misconceptions is present.

An explanation can be categorized as “Strong Evidence” of a misconception, “Weak Evidence” of a misconception, or “No Evidence” of a misconception.
Scoring

What counts as “Strong Evidence” of a misconception in the post-assessment?

In general, responses with strong evidence of a misconception include a clear indication that the student is exhibiting the reasoning typical for that misconception. There is no need to make inferences about what the student is thinking; that thinking is quite clear from the combination of the selected response and the explanation. Below are two examples of student responses with strong evidence of a misconception, using pre-assessment items. To see additional examples of student responses that illustrate these misconceptions, go to the “Sample Student Responses” on page 36.

Example A – Strong Evidence of M1

For M1, the explanation will include clear evidence that the student is adding the numerators and denominators as separate whole numbers. (For a more detailed description of this misconception, see the video explanations of each misconception.)

For item 4, this student chose 3/2, indicating the possibility of M1 thinking. In addition, the explanation provides clear evidence that the student is simply subtracting the numerators and subtracting the denominators.

Example B – Strong Evidence of M2

For M2, the explanation will include clear evidence that the student is finding a common denominator, not changing the numerators, then adding or subtracting. (For a more detailed description of this misconception, see the video explanations of each misconception.)

For item 6, this student has chosen 4/28, which indicates the possibility of M2 thinking. The explanation then clearly shows finding a common denominator (although the student’s multiplication is incorrect), and simply subtracting the existing numerators.
Scoring

Can a “Correct” response be considered to have “Strong Evidence?”

Yes, a correct response can also have “Strong Evidence,” “Weak Evidence,” or “No Evidence” as well. While it is not necessary to categorize correct responses as strong, weak or non-existent for the purposes of this diagnostic assessment, you may want to note this on your scoring template for your own purposes.

What counts as “Weak Evidence” of a misconception in the post-assessment?

Responses with weak evidence of a misconception include some indication that the student is exhibiting the reasoning typical for that misconception. However, these responses also generally require making more inferences about what the student was thinking, or they leave some question or doubt about whether the misconception is present or to what degree it is present. Below are two examples of student responses with weak evidence of a misconception, using pre-assessment items. To see additional examples of student responses that illustrate these misconceptions, go to the “Sample Student Responses” on page 36.

Example A – Weak Evidence of M1

For item 5, this student chose 4/1, which indicates the possibility of M1. In the explanation, the student simply repeats the problem. It is likely that they simply subtracted numerators and denominators, but because it is not clearly spelled out, this is considered “Weak Evidence” of M1.

Example B – Weak Evidence of M2

For item 2, this student chose 3/15, which indicates the possibility of M2. Again, in the explanation, the student simply repeats the problem. It is likely that they simply multiplied to find the common denominator, then just added the numerators, but because it is not clearly spelled out, this is considered “Weak Evidence” of M2.

What if the student selects one of the choices, but provides no explanation?

If a student selects an M1 or M2 response choice but provides no explanation at all, this is not considered convincing evidence of the misconception, and can be scored as “Other” on the scoring template.
Scoring

What if the student’s choice matches a misconception, but the explanation does not support it?

If a student’s response choice suggests a possible misconception, but the student’s explanation does not support it, then the item is not considered to be indicative of the misconception.

3. After you have analyzed each of the individual items for a student, use the guidelines below to determine whether the student has either of the misconceptions.

This diagnostic assessment has been validated to predict the possible presence of M1 or M2 for a student. If a student is determined to show evidence of the misconception on even just one of the items, the students is likely to have that misconception, regardless of whether the evidence is coded as “Strong” or “Weak.” The relative number of items with weak or strong evidence gives you information about how strongly the misconception may be present for the student.

What if my student has only one item indicating one of the misconceptions with “Weak Evidence,” and the rest are correct?

Even if your student has only one item with “Weak Evidence” of a misconception, this diagnostic assessment is validated to predict that it is likely your student may have that misconception. However, the presence of only one item with weak evidence suggests that the misconception may not be very deeply rooted in this student’s thinking.

You may want to keep an eye on this student during regular classwork to watch for other evidence of this misconception.

What if the student’s explanation is contradictory to the multiple-choice response chosen?

If you come across a response in which the explanation seems to contradict the response choice, it is considered a possible indication of the misconception. Look for additional evidence, either on these assessments or from the student’s comments in class.
(Optional) Scoring Practice Items—Post-Assessment

The following sample student responses are provided as an optional practice set. If you would like to practice scoring several items to further clarify your understanding of the scoring process, you may try scoring the following 10 items.

We recommend scoring one or two at a time and checking your scoring as you go against our key, found on page 32.

Practice Example 1

Show and explain your work here.

\[
\frac{5}{7} - \frac{1}{4} = \frac{4}{3} \\
\frac{13}{28} - \frac{7}{28} = \frac{6}{28} = \frac{3}{14}
\]

Practice Example 2

Show and explain your work here.

“I think 2/20 because I know that 4 x 5 = 20 and that 1 + 1 = 2.”

Practice Example 3

Show and explain your work here.

“I got 9/20 because first I found a common denominator which was 5 x 4 = 20 so now my problem was 5/20 + 4/20 and it equaled 9/20.”
Scoring

Practice Example 4

Show and explain your work here.

\[ \frac{4}{5} + \frac{1}{3} = \frac{7}{15} \]

\[ \frac{3}{15} \]

Practice Example 5

Show and explain your work here.

\[ 5 \times 3 = 15 \]
\[ \frac{5}{15} + \frac{6}{15} = \frac{11}{15} \]

\[ 21 = 3 \]
\[ 5 + 3 = 8 \]

Practice Example 6

Show and explain your work here.

\[ \frac{2}{5} + \frac{1}{3} = \frac{11}{15} \]

\[ \frac{3}{15} \]

Practice Example 7

Show and explain your work here.

“I simplified the denominators and multiplied the numerators [numerators] with the same number I simplified with. \( \frac{2}{5} + \frac{1}{3} = \frac{11}{15} \)
**Scoring**

Practice Example 8

4.

\[
\begin{align*}
\frac{4}{5} - \frac{1}{3} &= \frac{12}{15} - \frac{5}{15} = \frac{7}{15} \\
\frac{3}{2} &= \frac{15}{10} \\
\end{align*}
\]

Show and explain your work here.

\[
\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15} \\
\frac{3}{2} = \frac{15}{10}
\]

Practice Example 9

5.

\[
\begin{align*}
\frac{5}{6} - \frac{1}{5} &= \frac{25}{30} - \frac{6}{30} = \frac{19}{30} \\
\frac{4}{1} &= \frac{24}{30} \\
\end{align*}
\]

Show and explain your work here.

\[
\frac{5}{6} - \frac{1}{5} = \frac{25}{30} - \frac{6}{30} = \frac{19}{30} \\
\frac{4}{1} = \frac{24}{30}
\]

Practice Example 10

3.

\[
\begin{align*}
\frac{3}{8} + \frac{4}{5} &= \frac{15}{40} + \frac{32}{40} = \frac{47}{40} \\
\frac{7}{13} &= \frac{28}{40} \\
\end{align*}
\]

Show and explain your work here.

\[
\frac{3}{8} + \frac{4}{5} = \frac{15}{40} + \frac{32}{40} = \frac{47}{40} \\
\frac{7}{13} = \frac{28}{40}
\]
SCORING PRACTICE ITEMS ANSWER KEY—POST-ASSESSMENT

Practice Example 1

This is an example of M1 with “Strong Evidence.” The student does not write out his or her steps to show subtracting the numerators or denominators; however, given the three choices, there is little doubt that the student is simply subtracting numerators and denominators.

Practice Example 2

This is an example of M2 with “Strong Evidence.” The student chooses the M2 selected response (2/20), and then clearly shows multiplying the denominators to get 20 and adding the numerators to get 2.

Practice Example 3

This is a “Correct” example with “Strong Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response (9/20), shows how to find the common denominator and the equivalent fractions, and adds correctly.
Scoring

Practice Example 4

This is a “Correct” example with “Weak Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response \(7/15\), however the explanation suggests M1 thinking when the student subtracts the numerators and denominators. Since the student selected the correct response, but does not provide convincing evidence that he or she understands how to subtract fractions, this is considered “Weak Evidence.”

Practice Example 5

This is an example of M1 with “Weak Evidence.” The student selects the M1 response \(3/8\) but in the explanation, finds a common denominator, finds equivalent fractions, and adds correctly. The explanation also shows adding numerators and denominators, so it is unclear why the student finally chose \(3/8\).

Practice Example 6

This is a “Correct” example with “Strong Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response \(11/15\), shows how to find the common denominator and the equivalent fractions, and adds correctly. It appears the student is trying to use an area model to add the fractions as well, though the students’ use of that model is unclear.
Scoring

Practice Example 7

This is a “Correct” example with “Weak Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response (11/15), but leaves it unclear exactly how he or she arrived at 11/15. This makes it “Weak Evidence.”

Practice Example 8

This is an example of M2 with “Strong Evidence.” The student selects the M2 response (3/15), and clearly shows finding a common denominator of 15, then subtracting the numerators without first finding equivalent fractions.

Practice Example 9

This is a “Correct” example with “Strong Evidence” (though making any distinction between strong and weak correct responses is not necessary for this diagnostic assessment, and only for your own information about your student). The student selects the correct response (19/30), then clearly shows finding a common denominator, finding equivalent fractions, and subtracting correctly.
Scoring

Practice Example 10

This is an example of M1 with “Weak Evidence.” The student selects the M1 response (7/13). However, the explanation shows both adding numerators and denominators as well as finding a common denominator and equivalent fractions. Because it is unclear why the student has both ways of thinking about adding the fractions, it is considered “Weak Evidence” of M1.
Sample Student Responses

Review examples of student responses to assessment items.

The Fractions: Adding and Subtracting diagnostic assessment focuses on two particular misconceptions that students have regarding how to add and subtract fractions. Sample student responses indicative of each misconception are provided separately below, along with samples of correct student responses. To determine the degree of understanding and misunderstanding, it’s important to consider both the student’s answer to the selected response and the student’s explanation text and representations.

Misconception 1 (M1): Adding Numerators and/or Adding Denominators. Students sometimes apply whole-number reasoning by adding either the denominators or the numerators to determine the sum. (For more information, go to “Student Misconceptions” on page 3.)

The following student responses show examples of this misconception.

<table>
<thead>
<tr>
<th>Item</th>
<th>Sample Student Responses with Evidence of Misconception 2</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Assmt #2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$</td>
<td>$i\frac{added}{added} 2 + 1 = 3 \text{ and } 3 + 4 = 7$ and got 3/7.</td>
<td>• The M1 misconception selected-response is chosen AND</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• The explanation indicates the student is adding numerators together and adding denominators together.</td>
</tr>
<tr>
<td>Pre-Assmt #1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$</td>
<td>$\frac{1}{3} + \frac{1}{4} = \frac{2}{7}$ If you add up $\frac{1}{3}$ and $\frac{1}{4}$, you would get a fraction of $\frac{2}{7}$.</td>
<td>• The M1 misconception selected-response is chosen AND</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• The explanation indicates the student is adding numerators together and adding denominators together.</td>
</tr>
</tbody>
</table>
**Sample Student Responses**

**Pre-Assmt #6**

Show and explain your work here.

\[
\frac{3}{8} - \frac{1}{3} = \frac{9}{24} - \frac{8}{24} = \frac{1}{24}.
\]

So you do is 3 - 1 and you get 2 and then you do 8 - 3 and you get 5 so your answer is \(\frac{2}{5}\).

**Pre-Assmt #4**

Show and explain your work here.

\[
\frac{3}{4} - \frac{1}{6} = \frac{9}{24} - \frac{4}{24} = \frac{5}{24}.
\]

Because 3 - 1 = 2 and 6 - 4 = 2.

**Post-Assmt #3**

Show and explain your work here.

\[
\frac{3}{8} + \frac{4}{5} = \frac{15}{40} + \frac{32}{40} = \frac{47}{40}.
\]

\[
\frac{3}{8} + \frac{4}{5} = \frac{7}{13}.
\]

**Comments**

- The M1 misconception selected-response is chosen
- The explanation indicates the student is subtracting numerators as well as denominators.

- The M1 misconception selected response is chosen
- The explanation indicates the student is subtracting numerators as well as denominators.

- The M1 misconception selected-response is chosen
- The explanation indicates the student is adding numerators together and adding denominators together.
Sample Student Responses

**Misconception 2 (M2): Common Denominators with Incorrect Numerators.** Students sometimes apply a partial algorithm approach where they multiply the denominators but fail to change the numerators to maintain equivalence. (For more information, go to “Student Misconceptions” on page 3)

<table>
<thead>
<tr>
<th>Item</th>
<th>Sample Student Responses with Evidence of Misconception 3</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-Assmt #6</td>
<td><img src="image1.png" alt="Image" /></td>
<td>• The M2 misconception selected-response is chosen AND • The explanation explicitly shows the student finding a common denominator then subtracting the numerators.</td>
</tr>
<tr>
<td>Post-Assmt #3</td>
<td><img src="image2.png" alt="Image" /></td>
<td>• The M2 misconception selected-response is chosen AND • The explanation shows fractions with a common denominator, and the numerators are added together.</td>
</tr>
<tr>
<td>Pre-Assmt #1</td>
<td><img src="image3.png" alt="Image" /></td>
<td>• The M2 misconception selected-response is chosen AND • The explanation shows the student’s process of finding a common denominator, and the numerators are added together.</td>
</tr>
<tr>
<td>Pre-Assmt #3</td>
<td><img src="image4.png" alt="Image" /></td>
<td>• The M2 misconception selected-response is chosen AND • The student describes finding a common denominator, and adding the numerators.</td>
</tr>
</tbody>
</table>

I just added 2 + 3 and got 5 then I multiplied 9 x 5 and that equaled 45 so I picked the answer that said 5/45.
Correct Reasoning

Students with correct reasoning about comparing fractions are often able to do one or more of the following:

- calculate a common denominator
- create equivalent fractions
- correctly add or subtract only the numerators.

<table>
<thead>
<tr>
<th>Item</th>
<th>Sample Student Responses with Correct Reasoning</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Assmt#6</td>
<td><img src="image1.png" alt="Image" /></td>
<td>• The correct selected-response is chosen AND</td>
</tr>
<tr>
<td></td>
<td>I got 1/24 because 8 x 3 = 24 and 24 would be both of the fraction’s common denominator.</td>
<td></td>
</tr>
<tr>
<td>Pre-Assmt#1</td>
<td><img src="image2.png" alt="Image" /></td>
<td>• The correct selected-response is chosen AND</td>
</tr>
<tr>
<td>Post-Assmt#2</td>
<td><img src="image3.png" alt="Image" /></td>
<td>• The correct selected-response is chosen AND</td>
</tr>
</tbody>
</table>
**Sample Student Responses**

**Post-Assmt #1**

**1.** What I did was $\frac{1}{4} + \frac{1}{5}$. $5 \times 4 = 20$ and then I did $5 \times 1 = 5$ and $4 \times 1 = 4$, then my numbers were $\frac{5}{20} + \frac{4}{20} = \frac{9}{20}$.

- The correct selected-response is chosen
- The student's explanation shows how he or she calculated the common denominators. It also shows correct equivalent fractions, and correct computation.
Administering the Post-Assessment

Learn how to introduce the post-assessment to your students.

If the *Fractions: Adding and Subtracting* pre-assessment shows that any of your students have either or both of the misconceptions outlined in the Scoring Guide, plan and implement instructional activities designed to increase students’ understanding. The post-assessment provided here can then be used to determine if the misconception has been addressed.

Prior to Giving the Post-Assessment

- Arrange for 15 minutes of class time to complete the administration process, including discussing instructions and student work time. Since the post-assessment is designed to elicit a particular misconception after instruction, **you should avoid using or reviewing items from the post-assessment before administering it.**

Administering the Post-Assessment

- Inform students about the assessment by reading the following:

  *Today you will complete a short individual activity, which is designed to help me understand how you now think about adding and subtracting fractions, a topic we have been working on in class.*

- Distribute the assessment and read the following:

  *Like before, the activity includes 6 problems. For each problem, choose your answer by completely filling in the circle to show which answer you think is correct. Because the goal of the activity is to learn more about how you think about adding and subtracting fractions, it’s important for you to include some kind of explanation in the space provided. This can be a picture or words, or a combination of pictures and words that shows how you chose your answer.*

  *You will have about 15 minutes to complete all the problems. When you are finished, please place the paper on your desk and quietly [read, work on ___] until everyone is finished.*
Administering the Post-Assessment

- Monitor the students as they work on the assessment, making sure that they understand the directions. Although this is not a strictly timed assessment, it is designed to be completed within a 15-minute timeframe. Students may have more time if needed. When a few minutes remain, say:

  You have a few minutes left to finish the activity. Please use this time to make sure that all of your answers are as complete as possible. When you are done, please place the paper face down on your desk. Thank you for working on this activity today.

- Collect the assessments.

After Administering the Post-Assessment

Use the analysis process (found in the Scoring Guide PDF document under the Scoring Process section and found on page 7 of this document) to analyze whether your students have these misconceptions:

- **Misconception 1: Adding Numerators and/or Adding Denominators**
- **Misconception 2: Common Denominators with Incorrect Numerators**

Some students who previously had the misconception will no longer have it—the ideal case. Consider your instructional next steps for those students who still show evidence of the misconception.
### Scoring Guide Template – Fraction Addition and Subtraction

<table>
<thead>
<tr>
<th>Student:</th>
<th>Pre # 1</th>
<th>Pre # 2</th>
<th>Pre # 3</th>
<th>Pre # 4</th>
<th>Pre # 5</th>
<th>Pre # 6</th>
<th>Likelihood?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
</tr>
<tr>
<td></td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
</tr>
<tr>
<td>Post # 1</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
</tr>
<tr>
<td></td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
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<tr>
<td></td>
<td>Cor M1 M2</td>
<td>Other</td>
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<td>Other</td>
<td>Cor M1 M2</td>
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<tr>
<td></td>
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<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
</tr>
<tr>
<td>Post # 1</td>
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<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
<td>Cor M1 M2</td>
<td>Other</td>
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<td>Str Wk</td>
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<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
<td>N/A</td>
<td>Str Wk</td>
</tr>
</tbody>
</table>

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### Fractions

#### Adding and Subtracting

**Pre-Assessment**

**Directions:** Solve each problem. Select your answer from the choices provided.

1. \( \frac{1}{3} + \frac{1}{4} = \)
   - \( \frac{2}{7} \)
   - \( \frac{7}{12} \)
   - \( \frac{2}{12} \)

   Show and explain your work here.

2. \( \frac{2}{3} + \frac{1}{4} = \)
   - \( \frac{11}{12} \)
   - \( \frac{3}{12} \)
   - \( \frac{3}{7} \)

   Show and explain your work here.

3. \( \frac{2}{9} + \frac{3}{5} = \)
   - \( \frac{5}{45} \)
   - \( \frac{5}{14} \)
   - \( \frac{37}{45} \)

   Show and explain your work here.
## Fractions - Adding and Subtracting

### Pre-Assessment

<table>
<thead>
<tr>
<th></th>
<th>4.</th>
<th>Show and explain your work here.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{3}{4} - \frac{1}{6} = )</td>
<td>( \frac{2}{24} ) ( \frac{7}{12} ) ( \frac{2}{2} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>5.</th>
<th>Show and explain your work here.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{7}{8} - \frac{1}{4} = )</td>
<td>( \frac{6}{4} ) ( \frac{6}{32} ) ( \frac{5}{8} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>6.</th>
<th>Show and explain your work here.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{3}{8} - \frac{1}{3} = )</td>
<td>( \frac{1}{24} ) ( \frac{2}{5} ) ( \frac{2}{24} )</td>
</tr>
</tbody>
</table>
## Fractions

**Adding and Subtracting**

**Post-Assessment**

Directions: Solve each problem. Select your answer from the choices provided.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Show and explain your work here.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[ \frac{1}{4} + \frac{1}{5} = ]</td>
<td>□ [ \frac{2}{9} ] □ [ \frac{9}{20} ] □ [ \frac{2}{20} ]</td>
</tr>
<tr>
<td>2.</td>
<td>[ \frac{2}{5} + \frac{1}{3} = ]</td>
<td>□ [ \frac{11}{15} ] □ [ \frac{3}{15} ] □ [ \frac{3}{8} ]</td>
</tr>
<tr>
<td>3.</td>
<td>[ \frac{3}{8} + \frac{4}{5} = ]</td>
<td>□ [ \frac{7}{40} ] □ [ \frac{7}{13} ] □ [ \frac{47}{40} ]</td>
</tr>
</tbody>
</table>
### Fractions - Adding and Subtracting

#### Post-Assessment

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>( \frac{4}{5} - \frac{1}{3} = )</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>( \frac{3}{15} )</td>
<td></td>
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<td></td>
<td></td>
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<tr>
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<td>( \frac{7}{15} )</td>
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<td>( \frac{3}{2} )</td>
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<tr>
<td></td>
<td>Show and explain your work here.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 5. | \( \frac{5}{6} - \frac{1}{5} = \) |   |   |   |   |   |
|   |   |   |   |   |   |   |
|   | \( \frac{4}{1} \) |   |   |   |   |   |
|   | \( \frac{4}{30} \) |   |   |   |   |   |
|   | \( \frac{19}{30} \) |   |   |   |   |   |
|   | Show and explain your work here. |   |   |   |   |   |

| 6. | \( \frac{5}{7} - \frac{1}{4} = \) |   |   |   |   |   |
|   |   |   |   |   |   |   |
|   | \( \frac{13}{28} \) |   |   |   |   |   |
|   | \( \frac{4}{3} \) |   |   |   |   |   |
|   | \( \frac{4}{28} \) |   |   |   |   |   |
|   | Show and explain your work here. |   |   |   |   |   |

*Eliciting Mathematical Misconceptions*